# Prediction of the Visible Signature of Solid Rocket Plumes

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**Analytic expressions are presented defining the visible contrast of a rocket plume against the sky produced by scattering of sunlight and sky light by primary and secondary smoke in the plume. Allowances are made for cloud cover and atmospheric attenuation. To use these equations for prediction of the probability of detection by an observer, the rocket plume and the formation of smoke must first be described by existing computer models. A correlation available in the literature is used for prediction of the probability of detection as a function of contrast and visual angle. Required propellant data include the number of particles produced per unit mass of propellant by combustion, their size distribution, and their composition. The analytical procedure is encoded in a computer program, VISIG, suitable for use on a microcomputer. An example is given for the prediction of the probability of detection by an observer of a rocket flying at 5000 ft when viewed against the sky for a variety of sun, sky, and atmospheric conditions.**

## Nomenclature

$a_v$ = visual angle in arc-min
$B$ = brightness of the background transmitted through the plume
$B_o$ = brightness of the background
$B_p$ = brightness of the plume
$B_{sun}$ = brightness of the scattered sunlight
$B_{sky}$ = brightness of the scattered sky light
$C$ = visual threshold contrast for 50% probability of detection
$C_A$ = average plume contrast
$C_p$ = local plume contrast
$C_{Ap}$ = average apparent plume contrast after attenuation
$D$ = diameter of the plume
$(i_1 + i_2)$ = the Mie intensity function—a function of index of refraction, particle or droplet radius, wavelength of light, and angle of scatter
$I_s/B_o$ = sunlight-to-background brightness factor—a function of cloud-cover conditions
$K_s$ = Mie extinction coefficient—a function of index of refraction, particle or droplet radius, and wavelength of light
$K_{1/2}$ = correlation constant in Eq. (7)
$L$ = width of the plume through which the light passes
$m$ = correlation constant in Eq. (7)
$N$ = local number of particles per unit plume gas volume—a function of axial and radial plume position derived from code DROP
$P$ = probability of visual detection
$PSI$ = plume streamline
$R$ = range from the plume to the observer
$r$ = particle or droplet radius
$RBAR$ = average droplet radius
$X$ = distance into the plume from its edge
$Y$ = integrating variable plume radius
$Z$ = plume axial length
$\alpha,\beta$ = Weibull distribution coefficients—a function of axial and radial plume position computed by code DROP
$\theta$ = scattering angle
$\lambda$ = sunlight wavelength peaked for human eye sensitivity, 0.55 $\mu$
$\sigma$ = atmospheric attenuation coefficient

## Introduction

A SOLID rocket plume is visible to an observer primarily because of afterburning of exhaust gases in the atmosphere, the presence of primary smoke particles from the propellant and insulation, secondary smoke droplets formed by condensation of water with dissolved gases, or because of all three phenomena. The present paper is concerned with the daytime visible signature produced by the scattering of sun and sky radiation by primary and secondary smoke in the plume. An observer detects the presence of the rocket plume by its contrast against a background that is assumed to be the sky. Depending on the sky cover, the optical geometry of the plume, sun, and observer, the plume will appear to be bright, gray, or invisible. The greater the contrast, the greater the probability of detection will be.

A review of the subject of smoke, visible signal attenuation by the plume, plume visibility, and the probability of its detection may be found in Ref. 1. Hoshizaki et al.[2] developed a computer code, suitable for mainframe computers, for predicting the contrast and probability of detection of a rocket plume. Their code included routines for dealing with the multiple scattering of light by optically dense plumes. The code has found only limited applicability, possibly because of its complexity and costs of computer execution. Victor and Breil[3] and more recently Victor[4] devised a simplified method for predicting the visual contrast of a plume, due to scattering from primary alumina smoke, and the range at which the smoke trail disappears.

In the present paper, the complexities of multiple scattering of Ref. 2 have been avoided by limiting the computations to cases for which single scattering of light is satisfactory. In addition, some of the simplifications of Refs. 3 and 4 have been removed, and the analysis extended to include the effects of secondary smoke as well as primary smoke. The resulting computer program, VISIG, can be run on a microcomputer to predict the contrast against the sky and the probability of detection of a solid rocket plume by an observer.

## Background

To predict plume visibility, it is necessary to know the number density, size distribution, and chemical composition of the particles emitted from the rocket nozzle. Much of the particle data reported in the literature[5] were obtained with nozzle efficiency in mind, and emphasize the larger particles in which most of the mass of the particles resides. The data

Received Jan. 17, 1989; revision received June 28, 1989. 

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largely neglect the more numerous particles in the submicron size range that are more important to visibility considerations. Thorn and Pinkley[6] have collected a significant library of data on optical attenuation of light by primary and secondary smoke for various propellants and insulations. The information on the particulates for this paper were extracted from their optical measurements by a technique described in Ref. 7.

The predictions also require definition of the plume temperature, velocity, and gas composition fields. Programs such as LAPP[8] or SPF[9] can be used for this purpose. Having defined the plume, the code DROP[10] is used to predict the radial and axial number density, size distribution of primary and secondary smoke particles, and the composition of the secondary smoke droplets in the plume. The output from DROP is input to VISIG. The range, sky conditions, sun angle, atmospheric attenuation factor, and angle of the plume with the zenith complete the requirements for a particular computation.

## Theory

A review by Jarman and de Turville[11] of the prediction of contrast and visibility of chimney plumes has been extended to the case of a rocket plume. Local plume contrast is defined by the relationship

$$C_p = (B_p - B_o)/B_o = B_{\rm sun}/B_o + B_{\rm sky}/B_o + B/B_o - 1 \quad (1)$$

$$B_{\rm sun}/B_o = \tfrac{1}{2}(\lambda/2\pi)^2(I_s/B_o)\int_0^L\int_0^\infty Nf(r)(i_1 + i_2)\,{\rm d}r{\rm d}X \quad (2)$$

and for a sky of uniform brightness,

$$B_{\rm sky}/B_o = (\lambda/2\pi)^2\int_0^L\int_0^\infty\int_0^{\pi/2} Nf(r)(i_1 + i_2)\sin\theta\,{\rm d}\theta{\rm d}r{\rm d}X \quad (3)$$

$$(B/B_o - 1) = -\pi\int_0^L\int_0^\infty NK_sf(r)r^2{\rm d}r{\rm d}X \quad (4)$$

where $f(r) = \alpha\beta\, r^{\beta-1}\exp(-\alpha r^\beta)$, the plume particle radius Weibull density distribution—a function of plume radial and axial position.

Details of the Mie functions for light scattering are given in van der Hulst.[12] The values of the extinction coefficient and the intensity functions are derived from Dave.[13]

The average contrast for the length of the plume is computed from

$$C_A = (2/ZD)\int_0^Z\int_0^{D/2} C_p\,{\rm d}Y{\rm d}X \quad (5)$$

The plume contrast is attenuated by the atmosphere. The average apparent plume contrast after attenuation may be written as

$$C_{Ap} = C_A/\{1 + [\exp(\sigma R) - 1]\} \quad (6)$$

Typical values of $\sigma$ are given for various atmospheric conditions in Ref. 14.

Hoshizaki et al.[2] approximate an adjusted visual threshold function relating the visual angle and the visual threshold contrast for 50% probability of detection, from data taken mostly from Refs. 15 and 16, in the form

$$a_vC^m = K_{1/2} \quad (7)$$

Values taken from their report are given in Table 1. The visual angle is based on the diameter of an equivalent circle (whose area is equal to the visible length times the average diameter of the plume) divided by the range. The effective length of the plume for visual detection is equal to the foveal limit of the eye, 0.029 rad or 99.69 arc-min.

**Table 1 Adjusted visual threshold correlation**

| Target contrast | $m$ | $K_{1/2}$ |
|---|---|---|
| $0.02 < C < 0.06$ | 1.23 | 0.33 |
| $0.06 < C < 1.0$ | 0.56 | 1.5 |

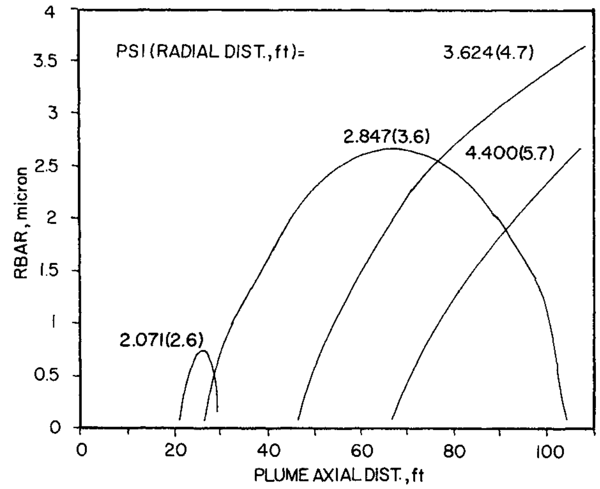


**Fig. 1 Secondary smoke prediction by DROP. *RBAR* (μm) vs plume axial distance (ft) with *PSI* (corresponding plume radius, ft) as parameter.**

For a 5000-ft range, for example, the visible length of the plume is 145 ft. If the average diameter of the plume is 14 ft or 0.0028 (9.63 arc-min), the visual angle of the equivalent circle is 34.96 arc-min. From Table 1, a contrast of 0.023 is calculated for a 50% probability of detection. The actual probability of visual detection is then calculated from

$$P = -0.106\,C_R + 0.909\,C_R^2 - 0.303\,C_R^3 \quad (8)$$

where $C_R = C_{Ap}/C$.

## Example

To demonstrate the use of VISIG[17] for visibility prediction, a case has been selected for a rocket flying horizontally at 250 fps, at an altitude of 5000 ft, directly overhead in daylight. The air temperature is 75°F and the relative humidity 95%. The sun is at 30 deg from the zenith and there is a light atmospheric haze. The rocket propellant comprises nominally 87 wt % ammonium propellant, 1 wt % aluminum, and 12 wt % rubber binder. The probability of detection of the rocket is to be predicted as a function of the sky conditions.

Some of the output from the prediction of secondary smoke by DROP is shown graphically in Fig. 1, where the average droplet radius *RBAR* for several plume streamlines *PSI* is plotted as a function of axial distance from the nozzle exit. The secondary smoke generally forms at the outer radii of the plume where the local temperature and concentrations of condensable gases permit. The average plume radius in feet corresponding to each streamline where this occurs is noted in parentheses in the figure. As discussed in more detail in Ref. 1, only those particles on each streamline that are larger than a critical size nucleate the condensation of condensable gases to form secondary smoke droplets. The critical nucleus size for condensation is dictated by the plume local gas composition and temperature. If the local gas temperature is too high, the partial pressures of the condensable gases too low, or the nuclei too small, condensation will not take place. For a given streamline on which local conditions permit condensation and growth, the droplets grow to a relatively uniform size. Further downstream, the droplets may evaporate as heat diffuses to the streamline from the hotter center region. At far-field axial distances, the droplets on the outer streamlines tend to grow to a steady-state size and composition.

Output from DROP ($\alpha$, $\beta$, $N$, and droplet composition) for 11 axial positions and six streamlines (including the centerline) were input to VISIG. The sun scattering angle, plume zenith angle, mean wavelength of sunlight for eye sensitivity, sunlight-to-background brightness ratio $I_s/B_o$ (22.0, 5.5, 0.0 for a clear sky, sun behind a bright cloud, and an overcast sky, respectively), and an average atmospheric attenuation coefficient for a light haze, 0.2875 per km, also were used. Theoret-

Table 2 Probability of detection and plume contrast as a function of sky conditions

| $I_s/B_o$ | Axial distance, ft | Average contrast[a] | Apparent contrast[a] | Probability of detection, % |
|---|---|---|---|---|
| 22.0 | 50 | 1.33 | 0.861 | |
| | 70 | 1.66 | 1.07 | |
| | 90 | 2.37 | 1.53 | |
| | 110 | 2.96 | 1.91 | |
| | 130 | 5.45 | 3.52 | |
| | **Plume** | **2.50** | **1.62** | **100.0** |
| 5.5 | 50 | 0.300 | 0.194 | |
| | 70 | 0.233 | 0.150 | |
| | 90 | 0.456 | 0.294 | |
| | 110 | 0.444 | 0.286 | |
| | 130 | 1.01 | 0.650 | |
| | **Plume** | **0.420** | **0.271** | **100.0** |
| 2.25 | 50 | 0.128 | 0.0825 | |
| | 70 | −0.00557 | −0.00359 | |
| | 90 | 0.137 | 0.0883 | |
| | 110 | 0.0238 | 0.0154 | |
| | 130 | 0.267 | 0.172 | |
| | **Plume** | **0.0723** | **0.0466** | **38.3** |
| 2.20 | 50 | 0.0935 | 0.0603 | |
| | 70 | −0.0532 | −0.0343 | |
| | 90 | 0.0730 | 0.0471 | |
| | 110 | −0.0602 | −0.0388 | |
| | 130 | 0.119 | 0.0764 | |
| | **Plume** | **0.00281** | **0.00181** | **0.0** |
| 0.0 | 50 | −0.0444 | −0.0287 | |
| | 70 | −0.244 | −0.157 | |
| | 90 | −0.182 | −0.118 | |
| | 110 | −0.396 | −0.256 | |
| | 130 | −0.474 | −0.306 | |
| | **Plume** | **−0.275** | **−0.178** | **100.0** |

[a]Cross-sectional average and apparent contrasts presented for a given axial distance, and the overall averages for the length of the plume for the distance noted as **Plume** (145 ft).

ical values of $K_s$ and $(i_1 + i_2)$ were computed using the Dave[13] routine. The plume contrasts were then derived at four radial positions (symmetry of the plume is assumed) for each of five selected axial distances from the nozzle. The contrast was averaged over the cross section of the plume at each axial distance, and over the visual length of the plume at the 5000-ft range, 145 ft, for an average plume diameter of 14 ft. The average apparent plume contrast was determined at each axial distance and over the total visible length. Finally, from these results the probability of detection was deduced.

The results obtained for the series of selected sky conditions, as defined by $I_s/B_o$, are summarized in Table 2. The plume appears bright, dark (negative contrast), or invisible against the sky depending on the sky and atmospheric conditions. The probability of detection by the observer, 0%, 100%, or some intermediate value, likewise depends on the sky conditions and atmospheric attenuation.

## Discussion

No attempt has been made here to explore the range of choices of propellant and flight profile that would minimize the probability of detection. Detection is also highly dependent on the mass flux and diameter of the plume; for a given propellant, the lower the mass flux and the smaller the plume diameter, the lower the probability of detection. The plume temperature, the number density, and species of particulates and condensables strongly influence the formation of smoke, and subsequently the contrast and probability of detection.

The single-scattering computation limits the validity of the predictions to an optical depth of 0.40, equivalent to an extinction ratio of 0.67. Optical depths greater than 0.40 would likely increase the probability of detection. Considering the scatter in the correlation of probability of detection vs contrast,[15] and the other assumptions made in this analysis, use of single scattering is probably acceptable or at least realistic. The analytical procedure is most useful for propellant selection for those missions for which visibility is the prime consideration. The only experimental data needed for the comparisons are the nuclei data for the candidate propellants: the number of particles produced per unit mass of propellant by combustion, their size distribution, and composition.

The analytical visible signature model has not been confirmed by experimental measurements. However, predictions of secondary smoke and laser signal attenuation made with DROP, and another code, OSA, similar to subroutines incorporated in VISIG, have been found to be consistent with limited experimental flight data.[10] These data provide support, albeit incomplete, that the model is useful for prediction of rocket plume visibility.

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